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2008 J. Phys. A: Math. Theor. 41 304013

(http://iopscience.iop.org/1751-8121/41/30/304013)

IOP Publishing Journal of Physics A: Mathematical and Theoretical

J. Phys. A: Math. Theor. **41** (2008) 304013 (11pp) doi:10.1088/1751-8113/41/30/304013

The omega effect as a discriminant for spacetime foam

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Received 30 October 2007
Published 15 July 2008
Online at stacks.iop.org/JPhysA/41/304013

Abstract

If there is CPT violation, the nature of entanglement for neutral meson pairs produced in meson factories may, on general grounds, be affected. The new form of entanglement is the omega effect. Gravitational decoherence, due to spacetime foam, may be one route for deviations from CPT invariance. Two models of spacetime foam are considered. One, based on non-critical string theory, is able to produce the new correlations in a natural way. The other, based on the paradigm of thermal-like baths, is shown to be surprisingly resistant to producing the effect even on exercising a total freedom of choice for the state of the bath.

PACS numbers: 03.65.Yz, 03.65.Ud, 04.60.−m, 14.40.Aq

1. Introduction

The operator $\Theta = CPT$ where C is the charge conjugation operator, P is the parity operator and T the time reversal operator is a symmetry of the S-matrix for local, unitary and Lorentz invariant field theories [1]. If one of these assumptions is not valid then Θ may cease to be a symmetry or be ill-defined. We shall consider the situation where an otherwise relativistic theory is not unitary. The lack of unitarity at a fundamental level will be postulated as due to spacetime foam [2]. In the decoherence scenario the S-matrix of the effective low-energy field theory would then have to be replaced by a linear non-factorizable superscattering operator $\not{S}$ relating initial and final-state density matrices ρ [3]

$$\rho_{\text{out}} = \not{S}\, \rho_{\text{in}}. \tag{1.1}$$

If this is correct then the usual formulation of quantum mechanics has to be modified; from the theory of open systems (see e.g. [4, 5]) we know that, on tracing over the environmental degrees of freedom, a system can be described by a master equation

$$\partial_t \rho = \frac{\mathrm{i}}{\hbar}[\rho, H] + \Lambda\rho \tag{1.2}$$

where Λ is a Liouvillian superoperator. The foam can be regarded as an environment and so the open systems point of view is also a natural one. It should be pointed out that this suggestion may be invalid if there is *holography* in quantum gravity [6], such that any information on quantum numbers of matter, that at first sight appears to be lost into the horizon, is somehow reflected back from the horizon surface, thereby maintaining quantum coherence. This may happen, for instance, in some highly supersymmetric effective theories of strings [7], which however do not represent realistic low-energy theories of quantum gravity. Supersymmetry breaking complicates the issue, thus spoiling complete holography. Recently, Hawking, inspired by the above recent ideas in string theory, has also argued against the loss of coherence in an *Euclidean* quantum theory of gravity. In such a model, summation over trivial and non-trivial (black hole) spacetime topologies in the path over histories makes an asymptotic observer 'unsure' as to the existence of the microscopic black hole fluctuation thus resulting in no loss of quantum coherence. However, this sort of argument is plagued not only by the Euclidean formalism, with its concomitant problems of analytic continuation, but also by a lack of a concrete proof, at least up to now. Hence the hypothesis of non-unitary evolution deserves further investigation.

We shall consider two approaches to spacetime which represent distinct ontologies. The first, *D*-particle foam, depends on a picture and partial description in terms of capture and emission of stringy matter by *D*-particles [8] based on non-critical strings. At late times after this process the induced spacetime metric has interesting stochastic off-diagonal structure [9] due to *D*-particle recoil which has an effect on correlated meson flavour pairs compatible with the earlier conjectured ω-effect [10, 11]. The second picture of spacetime foam [12] that we will consider is based on a heuristic non-local effective theory approach to gravitational fluctuations. The non-locality arises because the fluctuation scale is taken to be intermediate between the Planck scale and the low-energy scale. Furthermore the dominant non-locality was assumed to be bilocal, and, from similarities to quantum Brownian motion [4, 13] plausible arguments can be given for a thermal bath model for spacetime foam [12].

It is rare to have a test which can qualitativley distinguish spacetime foams and we will show that the study of correlations in neutral flavoured mesons can provide one. Earlier work has suggested that this may be the case [11] but the analysis for the thermal bath case although suggestive was quite incomplete. In this paper, we will show that the two foams have qualitatively different behaviour even allowing for non-thermal states of the bath. Neutral mesons such as the *K*-mesons have in the past been pivotal in the study of discrete symmetries [14]. The decay of a (generic) meson (e.g. the ϕ-meson) with quantum numbers $J^{PC} = 1^-$ [15], leads to a pair state $|i\rangle$ of neutral mesons (M) which has the form of the entangled state

$$|i\rangle = \tfrac{1}{\sqrt{2}}(|\overline{M_0}(\overrightarrow{k})\rangle|M_0(-\overrightarrow{k})\rangle - |M_0(\overrightarrow{k})\rangle|\overline{M_0}(-\overrightarrow{k})\rangle). \tag{1.3}$$

This state has $CP = +$. If CPT is not defined then M_0 and $\overline{M_0}$ may not be identified and the requirement of $CP = +$ can be relaxed [10, 11]. Consequently, the state of the meson pair can be parametrized to have the form

$$\begin{aligned}|i\rangle = {}& (|\overline{M_0}(\overrightarrow{k})\rangle|M_0(-\overrightarrow{k})\rangle - |M_0(\overrightarrow{k})\rangle|\overline{M_0}(-\overrightarrow{k})\rangle)\\ &+\omega(|\overline{M_0}(\overrightarrow{k})\rangle|M_0(-\overrightarrow{k})\rangle + |M_0(\overrightarrow{k})\rangle|\overline{M_0}(-\overrightarrow{k})\rangle)\end{aligned} \tag{1.4}$$

where $\omega = |\omega|e^{i\Omega}$ is a complex CPT violating (CPTV) parameter [10]. It is useful at this stage to rewrite the state $|i\rangle$ in terms of the mass eigenstates. To be specific, from now on we shall restrict ourselves to the neutral Kaon system $K_0 - \overline{K}_0$, which is produced by a ϕ-meson at rest, i.e. $K_0 - \overline{K}_0$ in their C.M. frame. The CP eigenstates (on choosing a suitable phase convention for the states $|K_0\rangle$ and $|\overline{K_0}\rangle$) are, in standard notation, $|K_\pm\rangle$ with

$$|K_\pm\rangle = \tfrac{1}{\sqrt{2}}(|K_0\rangle \pm |\overline{K_0}\rangle). \tag{1.5}$$

The mass eigenstates $|K_S\rangle$ and $|K_L\rangle$ are written in terms of $|K_\pm\rangle$ as

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon_2|^2}}[|K_-\rangle + \varepsilon_2|K_+\rangle] \tag{1.6}$$

and

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon_1|^2}}[|K_+\rangle + \varepsilon_1|K_-\rangle] \tag{1.7}$$

where ε_i, $i = 1, 2$, measure the degree of CP symmetry breaking.

In terms of the mass eigenstates

$$|i\rangle \simeq \mathcal{C}\left\{\begin{array}{l}(|K_L(\overrightarrow{k})\rangle|K_S(-\overrightarrow{k})\rangle - |K_S(\overrightarrow{k})\rangle|K_L(-\overrightarrow{k})\rangle)+ \\ \omega(|K_S(\overrightarrow{k})\rangle|K_S(-\overrightarrow{k})\rangle - |K_L(\overrightarrow{k})\rangle|K_L(-\overrightarrow{k})\rangle)\end{array}\right\} \tag{1.8}$$

where $\mathcal{C} = \frac{\sqrt{(1+|\varepsilon_1|^2)(1+|\varepsilon_2|^2)}}{\sqrt{2}(1-\varepsilon_1\varepsilon_2)}$ [10] since ε_i are small. The states $|K_L(-\overrightarrow{k})\rangle$ and $|K_S(-\overrightarrow{k})\rangle$ can be considered to form a two-level system with

$$|K_L\rangle = |\uparrow\rangle \qquad |K_S\rangle = |\downarrow\rangle. \tag{1.9}$$

Similarly $|K_L(\overrightarrow{k})\rangle$ and $|K_S(\overrightarrow{k})\rangle$ form another two-level system. For convenience we will refer to the $|\uparrow\rangle$ and $|\downarrow\rangle$ as flavour in the rest of this paper. These two two-level systems will be labelled as $|\uparrow^{(i)}\rangle$ and $|\downarrow^{(i)}\rangle$ with $i = 1, 2$. The unnormalized state $|i\rangle$ will then be an example of a state

$$|\psi\rangle = |\uparrow^{(1)}\rangle|\downarrow^{(2)}\rangle - |\downarrow^{(1)}\rangle|\uparrow^{(2)}\rangle + \xi|\uparrow^{(1)}\rangle|\uparrow^{(2)}\rangle + \xi'|\downarrow^{(1)}\rangle|\downarrow^{(2)}\rangle. \tag{1.10}$$

In section 2, we will give some details for the D-particle foam and a perturbative analysis of the gravitationally dressed states. In section 3 similar details will be given for a model of thermal-like foam. We will demonstrate a qualitative difference between the two foams. For the thermal case the perturbative analysis is also backed up by a non-perturbative analysis in order to rule out any possible higher order ω-effect.

2. Gravitational decoherence from *D*-particle foam

The most mathematically-consistent model of quantum gravity to date appears to be *String Theory*, although other approaches, such as loop quantum gravity (based on a canonical formalism of quantization) are making progress [16]. The non-critical (Liouville) string [17] provides a formalism for dealing with decoherent quantum spacetime foam backgrounds that include microscopic quantum-fluctuating black holes [18]. Given the limited understanding of gravity at the quantum level, the analysis of modifications of the quantum Liouville equation implied by non-critical strings can only be approximate and should be regarded as circumstantial evidence in favour of a dissipative master equation governing evolution. In the context of two-dimensional toy black holes [19] and in the presence of singular spacetime fluctuations there are believed to be inherently unobservable delocalized modes which fail to decouple from light (i.e. the observed) states. The effective theory of the light states which are measured by local scattering experiments can be described by a non-critical Liouville string. This results in an irreversible temporal evolution in target space with decoherence and associated entropy production.

The following master equation for the evolution of stringy low-energy matter in a non-conformal σ-model can be derived [18],

$$\partial_t\rho = i[\rho, H] + :\beta^i \mathcal{G}_{ij}[g^j, \rho]:, \tag{2.1}$$

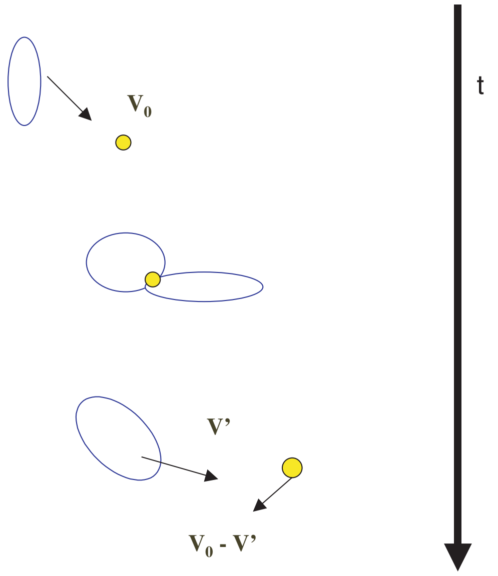

Figure 1. Illustration of the approach towards and capture of a closed string matter state by a *D*-particle. Subsequently a closed string is emitted by the *D*-particle.

where t denotes time (Liouville zero mode), H is the effective low-energy matter Hamiltonian, g^i are the quantum background target-space fields, β^i are the corresponding renormalization group β functions for scaling under Liouville dressings and $\mathcal{G}_{ij}$ is the Zamolodchikov metric [20, 21] in the moduli space of the string. The double colon symbol in (2.1) represents the operator ordering $: AB := [A, B]$. The index i labels the different background fields as well as spacetime. Hence the summation over i, j in (2.1) corresponds to a discrete summation as well as a covariant integration $\int \mathrm{d}^{D+1}y\sqrt{-g}$ where y denotes a set of $(D+1)$-dimensional target spacetime coordinates and D is the spacetime dimensionality of the original non-critical string. The discovery of new solitonic structures in superstring theory [22] has dramatically changed the understanding of target-space structure. These new non-perturbative objects are known as *D*-branes and their inclusion leads to a scattering picture of spacetime fluctuations. The study of *D*-brane dynamics has been made possible by Polchinski's realization [22] that such solitonic string backgrounds can be described in a conformally invariant way in terms of worldsheets with boundaries. On these boundaries Dirichlet boundary conditions for the collective target-space coordinates of the soliton are imposed.

Heuristically, when low-energy matter given by a closed (or open) string propagating in a $(D+1)$-dimensional spacetime collides with a very massive *D*-particle embedded in this spacetime, the *D*-particle recoils as a result (cf figure 1). Since there are no rigid bodies in general relativity the recoil fluctuations of the brane and their effectively stochastic back-reaction on spacetime cannot be neglected.

A concrete model in the form of a supersymmetric spacetime foam has been suggested in [23]. It is based on parallel braneworlds (with three large spatial dimensions), moving in a bulk spacetime which contains a 'gas' of *D*-particles. The number of parallel branes used is dictated by the requirements of target-space supersymmetry in the limit of zero-velocity branes. One of these branes represents allegedly our observable universe. As the brane moves in the bulk space, *D*-particles cross the brane in a random way. From the point of view of an observer in the brane the crossing *D*-particles will appear as spacetime defects which flash on and off, i.e. microscopic spacetime fluctuations. This will give the four-dimensional braneworld a '*D*-foamy' structure. Using this model for spacetime fluctuations we will obtain an expression for the *induced* spacetime distortion as a result of *D*-particle recoil. In the weakly coupled string limit, using logarithmic conformal field theory, it can be shown that [8]

$$g_{mn} = \delta_{mn}, \qquad g_{00} = -1, \qquad g_{0n} = \varepsilon(\varepsilon y_n + v_n t)\Theta_\varepsilon(t), \qquad m, n = 1, \ldots, D \tag{2.2}$$

where the suffix 0 denotes temporal (Liouville) components and $\Theta_\varepsilon(t)$ is a regularized Heavyside function with integral representation

$$\Theta_\varepsilon(t) = \frac{1}{2\pi \mathrm{i}} \int_{-\infty}^{\infty} \frac{\mathrm{d}q}{q - \mathrm{i}\varepsilon} \, \mathrm{e}^{\mathrm{i}qt}, \qquad v_n = (k_0 - k')_n, \tag{2.3}$$

with $k_0(k')$ the momentum of the propagating closed-string state before (after) the recoil; y_n are the spatial collective coordinates of the *D*-particle and ε^{-2} is identified with the target Minkowski time t for $t \gg 0$ after the collision [8] (in units where the string slope α' is taken to be 1). These relations have been calculated for non-relativistic branes where v_n is small. To leading order for large t,

$$g_{0n} \simeq \overline{v}_n \equiv \frac{v_n}{\varepsilon} \propto \frac{\Delta k_n}{M_P} \tag{2.4}$$

where Δk_n is the momentum transfer during a collision and M_P is the Planck mass (actually, to be more precise, $M_P = M_s/g_s$, where $g_s < 1$ is the (weak) string coupling and M_s is a string mass scale); so g_{0i} is constant in spacetime but depends on the energy content of the low-energy particle. The operator Δk_n is complicated to treat but on considering many collisions it can be regarded, in some sense, as random; consequently in [9] this transfer process was modelled by a classical Gaussian random variable r (for an isotropic foam) which multiplies the momentum operator $\widehat{p}$ for the particle:

$$\overline{u_i} \quad \rightarrow \quad \frac{r}{M_P}\widehat{p}. \tag{2.5}$$

Moreover the mean and variance of r are given by

$$\langle r \rangle = 0 \qquad \text{and} \qquad \langle r^2 \rangle = \sigma^2. \tag{2.6}$$

The process of capture and emission does not have to conserve flavour. Consequently we need to generalize the stochastic structure to allow for this. The fluctuations of each component of the metric tensor $g^{\alpha\beta}$ will then not be just given by the simple recoil distortion (2.4), but instead will be taken to have a 2×2 ('flavour') structure [11]:

$$\begin{aligned} g^{00} &= (-1 + r_4)\mathsf{1} \\ g^{01} &= g^{10} = r_0\mathsf{1} + r_1\sigma_1 + r_2\sigma_2 + r_3\sigma_3 \\ g^{11} &= (1 + r_5)\mathsf{1} \end{aligned} \tag{2.7}$$

where $\mathsf{1}$ is the identity and σ_i are the Pauli matrices. The above parametrization has been taken for simplicity and we will also consider motion to be in the x-direction which is natural since the meson pairs move collinearly. In any given realization of the random variables for a gravitational background, the system evolution can be considered to be given by the Klein–Gordon equation for a two-component spinless neutral meson field $\Phi = \binom{\phi_1}{\phi_2}$ (corresponding to the two flavours) with mass matrix $m = \frac{1}{2}(m_1 + m_2)\mathsf{1} + \frac{1}{2}(m_1 - m_2)\sigma_3$. We thus have

$$(g^{\alpha\beta} D_\alpha D_\beta - m^2)\Phi = 0 \tag{2.8}$$

where D_α is the covariant derivative. Since the Christoffel symbols vanish (as a_i are independent of spacetime) D_α coincide with ∂_α. Hence within this flavour changing background (2.8) becomes

$$\left(g^{00}\partial_0^2 + 2g^{01}\partial_0\partial_1 + g^{11}\partial_1^2\right)\Phi - m^2\Phi = 0. \tag{2.9}$$

We shall consider the two particle tensor product states made from the single-particle states $|\uparrow^{(i)}\rangle$ and $|\downarrow^{(i)}\rangle i = 1, 2$. This includes the correlated states which include the state of the ω-effect. In view of (2.9) the evolution of this state is governed by a Hamiltonian $\widehat{H}$

$$\widehat{H} = g^{01}(g^{00})^{-1}\widehat{k} - (g^{00})^{-1}\sqrt{(g^{01})^2k^2 - g^{00}(g^{11}k^2 + m^2)} \tag{2.10}$$

which is the natural generalization of the standard Klein–Gordon Hamiltonian in a one-particle situation where $\widehat{k}|\pm k, \alpha\rangle = \pm k|\pm k, \alpha\rangle$ with $\alpha = \uparrow$ or $\downarrow$. $\widehat{H}$ is a single-particle Hamiltonian and in order to study two particle states associated with $i = 1, 2$ we can define $\widehat{H}_i$ in the natural way (in terms of $\widehat{H}$) and then the total Hamiltonian is $\mathcal{H} = \sum_{i=1}^{2} \widehat{H}_i$. The effect of spacetime foam on the initial entangled state of two neutral mesons is conceptually difficult to isolate, given that the meson state is itself entangled with the bath. Nevertheless, in the context of our specific model, which is written as a stochastic Hamiltonian, one can estimate the order of the associated ω-effect of [11] by applying non-degenerate perturbation theory to the states $|\uparrow^{(i)}\rangle, |\downarrow^{(i)}\rangle, i = 1, 2$ (where the label $\pm k$ is redundant since i already determines it). Although it would be more rigorous to consider the corresponding density matrices, traced over the unobserved gravitational degrees of freedom, in order to obtain estimates it will suffice formally to work with single-meson state vectors).

Owing to the form of the Hamiltonian (2.10) the gravitationally perturbed states will still be momentum eigenstates. The dominant features of a possible ω-effect can be seen from a term $\widehat{H_I}$ in the interaction Hamiltonian

$$\widehat{H_I} = -(r_1\sigma_1 + r_2\sigma_2)\widehat{k} \tag{2.11}$$

which is the leading order contribution in the small parameters r_i (cf (2.7), (2.10)) in H (i.e. $\sqrt{\Delta_i}$ are small). In first order in perturbation theory the gravitational dressing of $|\downarrow^{(i)}\rangle$ leads to a state:

$$|\downarrow^{(i)}\rangle_{QG} = |\downarrow^{(i)}\rangle + |\uparrow^{(i)}\rangle\alpha^{(i)} \tag{2.12}$$

where

$$\alpha^{(i)} = \frac{\langle\uparrow^{(i)}|\widehat{H_I}|\downarrow^{(i)}\rangle}{E_2 - E_1} \tag{2.13}$$

and correspondingly for $|\uparrow^{(i)}\rangle$ the dressed state is obtained from (2.13) by $|\downarrow\rangle \leftrightarrow |\uparrow\rangle$ and $\alpha \to \beta$ where

$$\beta^{(i)} = \frac{\langle\downarrow^{(i)}|\widehat{H_I}|\uparrow^{(i)}\rangle}{E_1 - E_2}. \tag{2.14}$$

Here the quantities $E_i = \left(m_i^2 + k^2\right)^{1/2}$ denote the energy eigenvalues, and $i = 1$ is associated with the up state and $i = 2$ with the down state. With this in mind the totally antisymmetric 'gravitationally-dressed' state can be expressed in terms of the unperturbed single-particle states as

$$\begin{aligned}|\uparrow^{(1)}\rangle_{QG}|\downarrow^{(2)}\rangle_{QG} - |\downarrow^{(1)}\rangle_{QG}|\uparrow^{(2)}\rangle_{QG} &= |\uparrow^{(1)}\rangle|{-k}, \downarrow\rangle^{(2)} - |k, \downarrow\rangle^{(1)}|{-k}, \uparrow\rangle^{(2)} \\ &\quad + |\downarrow^{(1)}\rangle|\downarrow^{(2)}\rangle(\beta^{(1)} - \beta^{(2)}) + |\uparrow^{(1)}\rangle|\uparrow^{(2)}\rangle(\alpha^{(2)} - \alpha^{(1)}) \\ &\quad + \beta^{(1)}\alpha^{(2)}|\downarrow^{(1)}\rangle|\uparrow^{(2)}\rangle - \alpha^{(1)}\beta^{(2)}|\uparrow^{(1)}\rangle|\downarrow^{(2)}\rangle.\end{aligned}$$

It should be noted that for $r_i \propto \delta_{i1}$ the generated ω-like effect corresponds to the case $\xi = \xi'$ in (1.10) since $\alpha^{(i)} = -\beta^{(i)}$, while the ω-effect of [10] (1.8) corresponds to $r_i \propto \delta_{i2}$ (and the generation of $\xi = -\xi'$) since $\alpha^{(i)} = \beta^{(i)}$. In the density matrix these cases can be distinguished by the off-diagonal terms.

On averaging the density matrix over the random variables r_i, we observe that only terms of order $|\omega|^2$ will survive, with the order of $|\omega|^2$ being

$$|\omega|^2 = \mathcal{O}\left(\frac{1}{(E_1 - E_2)}(\langle\downarrow, k|H_I|k, \uparrow\rangle)^2\right) = \mathcal{O}\left(\frac{\Delta_2 k^2}{(E_1 - E_2)^2}\right) \sim \frac{\Delta_2 k^2}{(m_1 - m_2)^2} \tag{2.15}$$

for the physically interesting case in which the momenta are of order of the rest energies.

Recalling (cf (2.4)) that the variance Δ_1 is of the order of the square of the momentum transfer (in units of the Planck mass scale M_P) during the scattering of the single-particle state off a spacetime foam defect, i.e. $\Delta_1 = \zeta^2 k^2 / M_P^2$, where ζ is at present a phenomenological parameter. It cannot be further determined due to the lack of a complete theory of quantum gravity, which would in principle determine the order of the momentum transfer. We arrive at the following estimate of the order of ω in this model of foam:

$$|\omega|^2 \sim \frac{\zeta^2 k^4}{M_P^2 (m_1 - m_2)^2}. \tag{2.16}$$

Consequently for neutral kaons, with momenta of the order of the rest energies $|\omega| \sim 10^{-4}|\xi|$, whilst for B-mesons we have $|\omega| \sim 10^{-6}|\zeta|$. For $1 > \zeta \geqslant 10^{-2}$ these values for ω are not far below the sensitivity of current facilities, such as DAΦNE, and ζ may be constrained by future data owing to possible improvements of the DAΦNE detector or a B-meson factory. If the universality of quantum gravity is assumed then ζ can also be restricted by data from other sensitive probes, such as terrestrial and extraterrestrial neutrinos [24].

3. Thermal bath model for foam

Garay [12] has argued that the effect of non-trivial topologies related to spacetime foam and a nonzero minimum length can be modelled by a field theory with non-local interactions on a flat background in terms of a complete set of local functions $\{h_j(\phi, x)\}$ of the fields ϕ at a spacetime point x. His argument is based on general arguments related to problems of measurement [13]. Also, by considering the infinite redshifting near the horizon for an observer far away from the horizon of a black hole, Padmanabhan [25] has argued that a foam consisting of virtual black holes would magnify Planck scale physics for observers asymptotically far from the horizon and thus an effective non-local field theory description would be appropriate. The non-local action I can be written in terms of a sum of non-local terms I_n where

$$I_n = \frac{1}{n!}\int \mathrm{d}x_1 \cdots \int \mathrm{d}x_n\, f^{i_1\cdots i_n}(x_1, \ldots, x_n) h_{i_1}(x_1) h_{i_2}(x_2) \cdots h_{i_n}(x_n) \tag{3.1}$$

(where a summation convention for the indices has been assumed; the $f^{i_1\cdots i_n}$ depend on relative coordinates such as $x_1 - x_2$ and are expected to fall off for large separations, the scale of fluctuations being $l_* > \frac{1}{M_P}$). Assuming a form of weak coupling approximation it was further argued [12] that the retention of only the $n = 2$ term would be a reasonable approximation. Formally (i.e. ignoring the validity of Euclidean to Minkowski Wick rotations) the resulting non-locality can be written in terms of an auxiliary field φ through the functional identity

$$\begin{aligned}\exp&\left(\mathrm{i}\int \mathrm{d}x_1 \int \mathrm{d}x_2 f^{i_1 i_2}(x_1 - x_2) h_{i_1}(x_1) h_{i_2}(x_2)\right)\\ &= \int \mathrm{d}\varphi \exp\left(-\iint \mathrm{d}x_1\, \mathrm{d}x_2 k_{i_1 i_2}(x_1 - x_2)\varphi^{i_1}(x_1)\varphi^{i_1}(x_2)\right)\exp\left(\mathrm{i}\int \mathrm{d}x \varphi^j(x) h_j(x)\right)\end{aligned} \tag{3.2}$$

where $k_{i_1 i_2}$ is the inverse of $f^{i_1 i_2}$. The bilocality is now represented as a local field theory for ϕ subjected to a stochastic field φ. As shown in [26] and [27] this stochastic behaviour results [12] in a master equation of the type found in quantum Brownian motion, i.e. unitary evolution supplemented by diffusion. There is no dissipation which might be expected from the fluctuation diffusion theorem because the noise is classical. Since the energy scales for typical experiments are much smaller than those associated with gravitational quantum fluctuations Garay considered $f^{i_1 i_2}(x_1 - x_2)$ to be proportional to a Dirac delta function and argued that the master equation was that for a thermal bath.

A thermal field represents a bath about which there is minimal information since only the mean energy of the bath is known, a situation which maybe valid also for spacetime foam. In applications of quantum information it has been shown that a system of two qubits (or two-level systems) initially in a separable state (and interacting with a thermal bath) can actually be entangled by such a single-mode bath [28]. As the system evolves the degree of entanglement is sensitive to the initial state. The close analogy between two-level systems and neutral meson systems, together with the modelling by a phenomenological thermal bath of spacetime foam, makes the study of thermal master equations an intriguing one for the generation of ω-terms. The Hamiltonian $\mathcal{H}$ representing the interaction of two such two-level 'atoms' with a single-mode thermal field [29] is

$$\mathcal{H} = \nu a^{\dagger} a + \frac{1}{2}\Omega\sigma_3^{(1)} + \frac{1}{2}\Omega\sigma_3^{(2)} + \gamma\sum_{i=1}^{2}\left(a\sigma_+^{(i)} + a^{\dagger}\sigma_-^{(i)}\right) \tag{3.3}$$

where a is the annihilation operator for the mode of the thermal field and the σs are again the Pauli matrices for the two-level systems (using the standard conventions). The harmonic oscillator operators a and $a^{\dagger}$ satisfy

$$[a, a^{\dagger}] = 1, \qquad [a^{\dagger}, a^{\dagger}] = [a, a] = 0. \tag{3.4}$$

The $\mathcal{H}$ here is quite different from the $\mathcal{H}$ of the D-particle foam of the last section. There are no classical stochastic terms at this level of description and also $\mathcal{H}$ is not separable. In the D-particle foam model the lack of separabillity came solely from the entangled nature of the initial unperturbed state. The thermal master equation comes from tracing over the oscillator degrees of freedom. We will however consider the dynamics before tracing over the bath because, although the thermal bath idea has a certain intuitive appeal, it cannot claim to be rigorous and so for attempts to find models for the ω-effect it behoves us to entertain also deviations from the thermal bath state of the reservoir.

An important feature of $\mathcal{H}$ in (3.3) is the block structure of subspaces that are left invariant by $\mathcal{H}$. It is straightforward to show that the family of invariant irreducible spaces $\mathcal{E}_n$ may be defined by $\left\{\left|e_i^{(n)}\right\rangle, i = 1, \ldots, 4\right\}$ where (in obvious notation, with n denoting the number of oscillator quanta)

$$\left|e_1^{(n)}\right\rangle \equiv |\uparrow^{(1)}, \uparrow^{(2)}, n\rangle, \qquad \left|e_2^{(n)}\right\rangle \equiv |\uparrow^{(1)}, \downarrow^{(2)}, n\rangle, \qquad \left|e_3^{(n)}\right\rangle \equiv |\downarrow^{(1)}, \uparrow^{(2)}, n\rangle \tag{3.5}$$

and

$$\left|e_4^{(n)}\right\rangle \equiv |\downarrow^{(1)}, \downarrow^{(2)}, n\rangle.$$

The total space of states is a direct sum of the $\mathcal{E}_n$. We will write $\mathcal{H}$ as $\mathcal{H}_0 + \mathcal{H}_1$ where

$$\mathcal{H}_0 = \nu a^{\dagger} a + \frac{\Omega}{2}\left(\sigma_3^{(1)} + \sigma_3^{(2)}\right)$$

and

$$\mathcal{H}_1 = \gamma\sum_{i=1}^{2}\left(a\sigma_+^{(i)} + a^{\dagger}\sigma_-^{(i)}\right).$$

n is a quantum number and gives the effect of the random environment. In our era the strength γ of the coupling with the bath is weak. We expect heavy gravitational degrees of freedom and so $\Omega \gg \nu$. It is possible to associate both thermal and highly non-classical density matrices with the bath state. We will investigate whether this freedom is enough to generate ω-type terms.

We will calculate the stationary states in $\mathcal{E}_n$, using degenerate perturbation theory, where appropriate. We will be primarily interested in the dressing of the degenerate states $|e_2^{(n)}\rangle$ and $|e_3^{(n)}\rangle$ because it is these which contain the neutral meson entangled state. In second-order perturbation theory the dressed states are

$$\left|\psi_2^{(n)}\right\rangle = \left|e_2^{(n)}\right\rangle - \left|e_3^{(n)}\right\rangle + O(\gamma^3) \tag{3.6}$$

with energy $E_2^{(n)} = (n+1)\nu + O(\gamma^3)$ and

$$\left|\psi_3^{(n)}\right\rangle = \left|e_2^{(n)}\right\rangle + \left|e_3^{(n)}\right\rangle + O(\gamma^3) \tag{3.7}$$

with energy $E_3^{(n)} = (n+1)\nu + \frac{2\gamma^2}{\Omega-\nu} + O(\gamma^3)$. It is $|\psi_2^{(n)}\rangle$ which can in principle give the ω-effect. More precisely we would construct the state $\mathrm{Tr}\left(\left|\psi_2^{(n)}\right\rangle\left\langle\psi_2^{(n)}\right|\rho_B\right)$ where ρ_B is the bath density matrix (and has the form $\rho_B = \sum_{n,m} p_{nn'}|n\rangle\langle n'|$ for suitable choices of $p_{nn'}$). To this order of approximation $|\psi_2^{(n)}\rangle$ cannot generate the ω-effect since there is no admixture of $|e_1^{(n)}\rangle$ and $|e_4^{(n)}\rangle$. However it is *a priori* possible that this may change when higher orders in γ. We can show that $|\psi_2^{(n)}\rangle = |e_2^{(n)}\rangle - |e_3^{(n)}\rangle$ to all orders in γ by directly considering the Hamiltonian matrix $\mathcal{H}^{(n)}$ for $\mathcal{H}$ within $\mathcal{E}_n$; it is given by

$$\mathcal{H}^{(n)} = \begin{pmatrix} \Omega + n\nu & \gamma\sqrt{n+1} & \gamma\sqrt{n+1} & 0 \\ \gamma\sqrt{n+1} & (n+1)\nu & 0 & \gamma\sqrt{n+2} \\ \gamma\sqrt{n+1} & 0 & (n+1)\nu & \gamma\sqrt{n+2} \\ 0 & \gamma\sqrt{n+2} & \gamma\sqrt{n+2} & (n+2)\nu - \Omega \end{pmatrix}. \tag{3.8}$$

We immediately note that

$$\mathcal{H}^{(n)} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} = (n+1)\nu \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \tag{3.9}$$

and so, to all orders in γ, the environment does *not* dress the state of interest to give the ω-effect; clearly this is independent of any choice of ρ_B. One cannot say of course that other more complcated models of 'thermal' baths may not display the ω-effect but clearly a rather standard model rejects quite emphatically the possibility of such an effect. This by itself is very interesting. It shows that the ω-effect is far from a generic possibility for spacetime foams. Just as it is remarkable that the commonplace 'thermal' bath cannot accommodate the ω-effect, it also remarkable that the *D*-particle foam model manages to do so very simply.

4. Conclusions

The magnitude of the ω-effect which may not be far from the sensitivity of immediate future experimental facilities, such as a possible upgrade of the DAΦNE detector or a B-meson factory. In this work we have discussed two classes of spacetime foam models, which are not inconceivable that may characterize realistic situations of the (still elusive) theory of quantum gravity. It is interesting to continue the search for more realistic models of quantum gravity, either in the context of string theory or in other approaches, such as the canonical approach or the loop quantum gravity, in order to search for intrinsic CPT Violating effects in sensitive matter probes. Moreover, the phenomenological approach of foam baths may be guided by consideration of hydrodynamical theories [30]. Detailed analyses of global data, including very sensitive probes such as high energy neutrinos, are the only way forward in order to obtain some clues on the elusive theory of quantum gravity. Decoherence, induced by quantum gravity, is not only ruled out at present, but indeed it may provide the link between theory and experiment in this elusive area of physics.

Acknowledgments

This work is partially supported by the European Union through the FP6 Marie Curie Research and Training Network *UniverseNet* (MRTN-CT-2006-035863). I would like to thank N E Mavromatos for stimulating discussions.

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